Determination of a confidence region utilizing the empirical model becomes a trivial task. It is obtained by solving equation 6 together with

$$\phi(k_1,k_2) = \phi_{\min}\left\{1 + \frac{p}{n-p} F_{5\%}(p,n-p)\right\} \tag{7}$$

and using the established relationships for $c_{r,\text{calc}}$. For this example, the analysis of confidence regions, detailed in Miller and Frenklach (1983), revealed certain features of the experimental data which were not detected originally.

### NOTATION

| | |
|---|---|
| $a$ | = coefficients of empirical Eqs. 4 |
| $c$ (or [ ]) | = species concentration |
| $F_{5\%}(p,n-p)$ | = upper 5% point of F distribution with $p$ and $n-p$ degrees of freedom |
| $f$ | = numerically defined solution for Eq. 1 |
| $k$ | = reaction rate constant |
| $\mathbf{k}$ | = vector of reaction rate constants |
| $n$ | = number of observed responses |
| $p$ | = number of adjustable parameters, 2 in this example |
| $S_i^r$ | = logarithmic sensitivity of $r$th resposne to $i$th rate constant |
| $t$ | = reaction time |

#### Greek Letters

| | |
|---|---|
| $\nu_{ij}$ | = generalized stoichiometric coefficient of $j$th species in $i$th reaction |
| $\phi$ | = objective function |
| $\phi_{\min}$ | = the minimum value of the objective function |

#### Subscripts

| | |
|---|---|
| $i$ | = reaction number |
| $j$ | = species number |
| $r$ | = response number |
| $t$ | = at time $t$ |

### LITERATURE CITED


Box, G. E. P., and G. A. Coutie, "Application of Digital Computers in the Exploration of Functional Relationships," *Proc. I.E.E.*, **103B** (Suppl. No. 1), 100 (1956).

Box, G. E. P., and N. R. Draper, "The Bayesian Estimation of Common Parameters from Several Responses," *Biometrika*, **52**, 355 (1965).

Box, G. E. P., and N. R. Draper, "Measures of Lack of Fit for Response Surface Designs and Predictor Variable Transformations," *Technometrics*, **24**, 1 (1982).

Box, G. E. P., and W. G. Hunter, "The Experimental Study of Physical Mechanisms," *Technometrics*, **7**, 23 (1965).

Box, G. E. P., W. G. Hunter, and J. S. Hunter, *Statistics for Experimenters*, Wiley, New York (1978).

Box, G. E. P., et al., "Some Problems Associated with the Analysis of Multiresponse Data," *Technometrics*, **15**, 33 (1973).

Frank, P. M., *Introduction to System Sensitivity Theory*, Academic Press, New York (1978).

Frenklach, M., "Modeling," *Chemistry of Combustion Reactions*, W. C. Gardiner, Jr., Ed., Ch. 7, Springer-Verlag (1984).

Lifshitz, A., K. Scheller, and A. Burcat, "Decomposition of Propane behind Reflected Shocks in a Single Pulse Shock Tube," *Recent Developments in Shock Tube Research: Proc. of Int. Shock Tube Symp.*, D. Bershader and W. Griffith, Eds., 690, Stanford University Press, Stanford, CA (1973).

Lifshitz, A., and M. Frenklach, "Mechanism of the High Temperature Decomposition of Propane," *J. Phys. Chem.*, **79**, 686 (1975).

Miller, D. L., and M. Frenklach, "Sensitivity Analysis and Parameter Estimation in Dynamic Modelling of Chemical Kinetics," *Int. J. Chem. Kinet.*, **15**, 677 (1983).


*Manuscript received Jan. 24, 1984; revision received Feb. 13, 1984, and accepted Feb. 13.*

---

# Predicting the Holdup in Two-Phase Bubble Upflow and Downflow Using the Zuber and Findlay Drift-Flux Model


**N. N. CLARK**

**Particle Analysis Center,**
**West Virginia University,**
**Morgantown, WV 26506**

**and**

**R. L. FLEMMER**

**Department of Chemical Engineering,**
**University of Natal,**
**Durban, South Africa**


## INTRODUCTION

Relatively recent development of the deep shaft reactor (Collins and Elder, 1980; Cox et al., 1980; Kubota et al., 1978) and interest in similar devices of smaller bore (Clark and Flemmer, 1983) have demanded a better understanding of two-phase downflow in pipes. DSRs operate as mass transfer devices using vertical bubble flow. Knowledge of the gas void fraction present in the pipe is essential for the prediction of hydrostatic head, a factor influencing mass transfer rate. Upward two-phase flow has received much attention in the literature, but downflow a good deal less. Both upflow and

N. N. Clark was formerly at the Department of Chemical Engineering, University of Natal, Durban, South Africa. Correspondence concerning this paper should be addressed to him.

downflow were investigated here in a 100 mm diameter pipe to determine whether a similar model was applicable in both configurations.

## THE DRIFT-FLUX MODEL

In vertical two-phase flow, the average gas void fraction in the pipe, $\epsilon$, cannot be simply equated to the volumetric flowing gas fraction, $W_g/(W_g + W_l)$, for two reasons. First, the bubbles rise in the liquid due to gravitational forces, so that the local velocity of the gas and liquid phases differs. Second, if the gas void fraction is distributed in the pipe such that a disproportionate amount of gas is in a region of flow with a greater or lesser velocity than the average velocity in the pipe, then the average gas velocity will be further modified. Typically both gas void distributions and velocity distributions have coincident maxima at the pipe center so that, on average, the gas is traveling faster than the liquid. Taking both of these factors into account, Zuber and Findlay (1964, 1965) presented a drift-flux model that has been accepted widely for the prediction of holdup and applied to various two-phase flow regimes in upflow (Nassos and Bankoff, 1967; Govier and Aziz, 1972; Ardron and Hall, 1980).

$$W_g/\epsilon = C_o(W_g + W_l) + (U_{gm}\epsilon)_{av}/\epsilon \qquad (1)$$

where $C_o$ is a constant accounting for the interaction of velocity and gas void distributions, and the drift velocity term containing $U_{gm}$ accounts for the local slip between the phases. In "churn-turbulent" bubbly upflow, where bubble interaction and waking are strong, $C_o$ has been found to assume values between 1.1 and 1.3 (Wallis, 1969; Nassos and Bankoff, 1967; Ishii and Grolmes, 1978) and the drift velocity term is equated to the bubble rise velocity in an infinite medium, $U_b$. The formula of Harmathy (1960) may be used to predict $U_b$ for churn turbulent flow (Zuber and Findlay, 1964; Govier and Aziz, 1972; Clark and Flemmer, 1984a).

$$U_b = 1.53[\sigma g/\rho]^{1/4} \qquad (2)$$
$$= 0.25 \text{ m/s for an air-water system}$$

There has been some dispute as to whether $C_o$ assumes the same value in up- and downflow in similar apparatus, or whether the value differs between the two configurations. Zuber et al. (1967) found $C_o$ to be constant over a wide range of velocities in up- and downflow of boiling Santowax R, and recent work by the authors (Clark and Flemmer, 1983b) has shown $C_o$ to assume a constant value in up- and downflow of 1.16 for air-water bubbly flow in a 50 mm pipe. Conversely, Bhaga and Weber (1972) found $C_o$ to vary between the cases of upflow and countercurrent flow at low liquid velocities, and Martin (1976) found that $C_o$ differed between up and down slug flow. Oshinowo and Charles (1974) observed that bubble profiles, and hence $C_o$, differed between the cases of up- and downflow. Brown et al. (1969), proposing a drift-flux model similar to that of Zuber and Findlay, implied that $C_o$ would be greater than unity in upflow, yet less than unity in downflow, due to an inversion of void profiles, but their analysis is open to question, as they were obliged to assume drift velocities as low as 12 cm/s to justify the theory in downflow. Lorenzi and Sotgia (1978) investigated up and down air-water flow and concluded not only that $C_o$ varied between the two configurations but within them as well. It is possible, however, that Lorenzi and Sotgia's results were for a bubble flow that was more ideal than churn-turbulent.

The disagreements illustrated above indicated that a thorough investigation was necessary to assess the application of the drift-flux model in narrow-bore DSR design.

## EXPERIMENT

A two-phase flow loop of 100 mm pipe was constructed and a vertical 4 m test section placed between two ball valves in the center of an 8 m vertical length of the loop. The valves were linked by a tie-rod of adjustable length and could be rapidly closed with a ram fed by high pressure air. These valves were accurately synchronized by means of a pressure balance technique, a novel method outlined elsewhere (Clark and Flemmer, 1984b). Water was made to flow either up or down the test section by means of valve gear in the loop, and flow was monitored using a British standard orifice. Air flow was measured by rotameters and introduced into the water stream 2 m vertically below the test section for upflow analysis and 2 m above the test section for downflow. The air was introduced through four 8 mm diameter, 500 mm long copper pipes, each drilled with fifty 1 mm holes. These were placed within the pipe, parallel to the axis, each 30 mm from the pipe center, so that in cross section of the pipe the four copper pipes were at the corner of a square.

The bubble rise velocity in an infinite medium, $U_b$, was determined by measuring the bubble rise velocity relative to the liquid, $U_{gl}$, for a range of gas voidages, and regressing to find $U_b$ as the value of $U_{gl}$ at zero voidage. Values of the bubble rise velocity were monitored in the following way. A bubble flow was set up and the ball values closed rapidly and simultaneously. Bubbles rose in the test section to form an ullage beneath the upper ball valve. A theoretical examination of such a system shows that the pressure in the ullage will continue to rise from the time the valves are shut to the time when the last bubble rises from the base of the section to the gas-liquid interface between the liquid and the ullage. This pressure rise was monitored using a sensitive pressure cell and high speed chart recorder to produce a plot of ullage pressure against time. The time taken for the bubbles initially at the base of the test section to rise to the interface, and hence the bubble rise velocity, were inferred from the pressure-time plot. The experiment was then repeated, changing the gas flow rate; in this way data points for $U_{gl}$ were obtained for a range of gas voidages in the test section. The rise velocity in an infinite continuum, $U_b$, was then determined by regressing on these points to predict the rise velocity as the voidage became very small.

The gas voidage in the section was determined by closing the valves on a two-phase flow, allowing the two phases to separate, and observing the height of the gas-liquid interface in a sight glass of small bore attached to the test section.

A 700 mm long full bore glass pipe was installed in the test section. This permitted checking that the system was in bubbly flow and that no slugs or, in the case of downflow, falling films were present.

## RESULTS

The flow fitted the visual description of churn-turbulent flow given by Zuber and Findlay (1965). Bubble sizes ranged from 1.5 to 5 mm in diameter, and occasional larger bubble caps were present. No well-developed slug flow was observed.

The rise velocity of bubbles relative to the liquid, $U_{gl}$, was determined for various voidages, and results are given in Figure 1. The decrease in bubble rise velocity with increasing voidage may be explained in terms of the last small bubbles rising to the interface in "ideal bubbly flow," but this trend is not relevant to the work here. A regression was performed on the results using the equation proposed by Wallis (1969)

$$U_{gl} = U_b(1 - \epsilon)^n \qquad (3)$$

thus yielding a value for the bubble rise velocity at zero voidage. Best fit was found for $n = 0.702$ and $U_b = 0.25$ m/s, which is in exact agreement with the equation of Harmathy (1960). This value

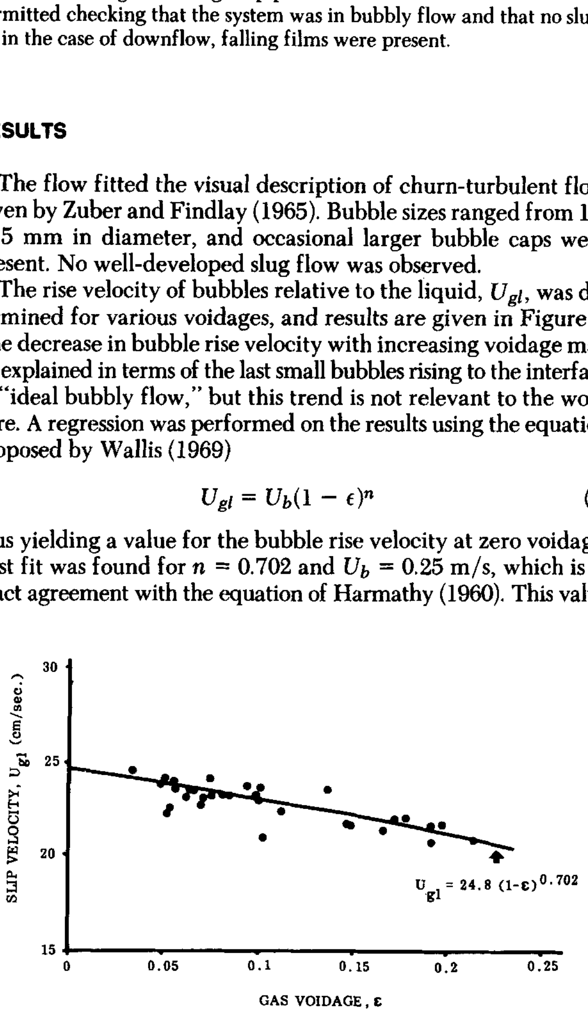


**Figure 1. Bubble slip velocity for a range of gas voidages (100 mm pipe).**

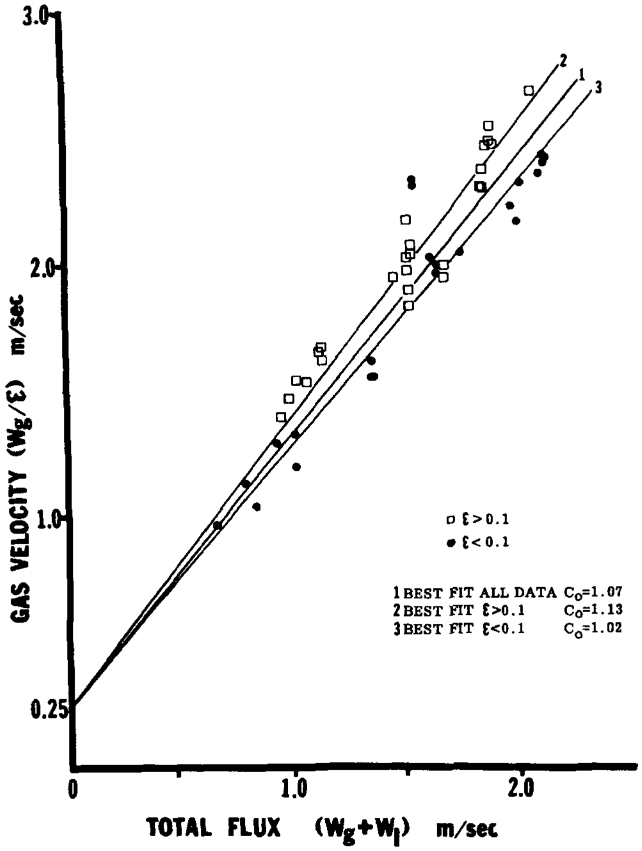


Figure 2. Drift-flux plot for upflow.

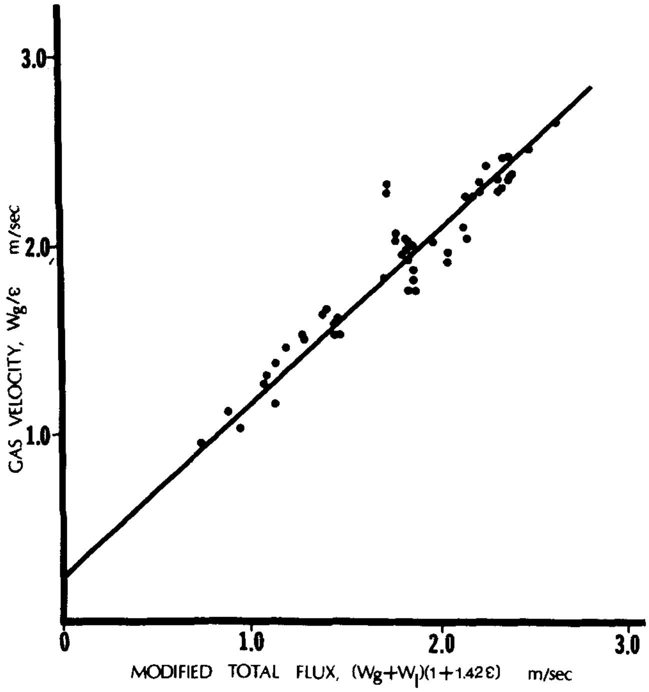


Figure 4. Modified Drift-flux plot for upflow.

of 0.25 m/s was then used for the drift velocity term in the Zuber and Findlay model, Eq. 1.

Equation 1 shows that the Zuber and Findlay drift-flux plot is best illustrated on a plot of the average gas velocity, $W_g/\epsilon$, versus the total superficial velocity, $W_g + W_l$. The data gained in up- and downflow are shown in Figures 2 and 3. Lines show regressions passed through the points under constraint of the rise velocity (Y intercept) of 0.25 m/s. Average value of $C_o$ in upflow was 1.07 and in downflow 1.17. Wide scatter of the points was observed and was found too great to be explained in terms of varying local slip. Local slip might, at worst, increase due to the presence of slugs. The slug rise velocity is given by the formula

$$U_{slug} = 0.35\,(gD)^{0.5}$$
$$= 0.34 \text{ m/s for an air-water system}$$

This would raise the drift velocity, which is 0.25 m/s for bubble flow, by only 0.09 m/s. Scatter in values of $W_g/\epsilon$, the gas velocity, is as broad as 0.6 m/s and so cannot be explained only by variation in the drift velocity term due to the presence of occasional slugs. Such scatter might be explained by variation in the profile constant, $C_o$. Examination of the results revealed a strong trend for $C_o$ to vary with voidage, although data scatter was still present in plots of $C_o$ versus gas voidage. The coefficients for the best linear variation of $C_o$ with voidage were found to be

Upflow: $C_o = 0.934(1 + 1.42\epsilon)$
Downflow: $C_o = 1.521(1 - 3.67\epsilon)$

All the results were finally illustrated by combining this linear

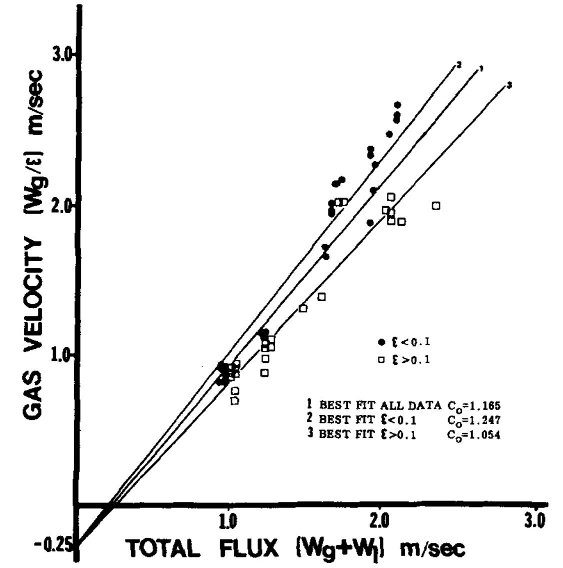


Figure 3. Drift-flux plot for downflow.

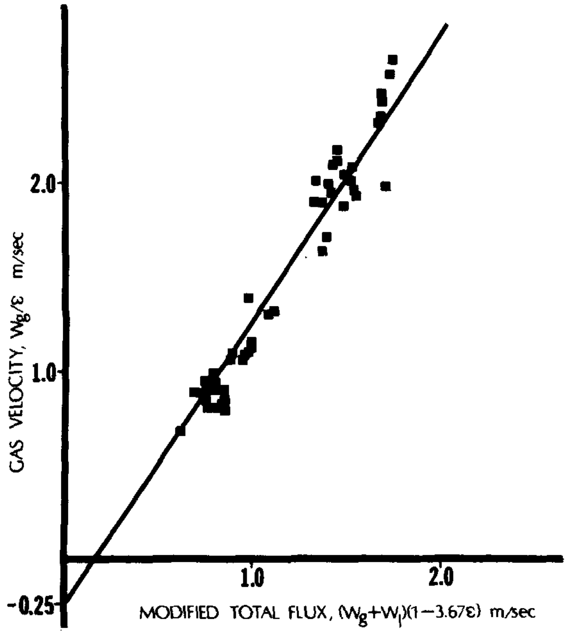


Figure 5. Modified drift-flux plot for downflow.

variation of $C_o$ with the conventional drift-flux plot, so that an equation of the form

$$W_g/\epsilon = C_l(1 + C_2\epsilon)(W_g + W_l) + U_b \qquad (4)$$

arose. The gas velocity, $W_g/\epsilon$, was then plotted against $(W_g + W_l)(1 + C_2\epsilon)$ to produce the modified plots with a slope of $C_1$, given in Figures 4 and 5. The linear relationships used here are not presented as a reliable correlation to describe the dependence of $C_o$ on the gas void fraction in all bubble flow situations, since the dependence of $C_o$ may vary for different pipe geometries.

## DISCUSSION

The variation of $C_o$ was in strong contrast to its constancy observed in the authors' 50 mm pipe work (Clark and Flemmer, 1984a), and may be attributed to a greater variation in mixture properties over the larger pipe diameter. Such variation supports the claims of Lorenzi and Sotgia (1978) and of Petrick and Kurdika (1966) that $C_o$ may vary, although the latter authors reported trends for the profile constant in upflow opposite to those observed here.

The variation of $C_o$ in upflow is best explained by the work of Serizawa et al. (1975) and Galaup and Delhaye (1976). They observed that at low voidages void profiles were saddle-shaped, with a large bubble population near the wall. Such saddle-shaped profiles have been observed by Herringe and Davis (1976) even after 108 pipe diameters of upflow. In this case there would certainly be no more than the average concentration of bubbles at the tube center, so that by the arguments presented above the interaction of void and velocity distributions would not be favorable, and low values of $C_o$ might ensue. With an increase in voidage, however, Serizawa et al. (1975) and Galaup and Delhaye (1976) observed that void profiles became more parabolic in shape, with a distinct maximum at the pipe center. This would lead to a high value of $C_o$ as the void and velocity distribution maxima coincided at the tube center.

A similar argument may be applied to downflow. Oshinowo and Charles (1974) observed in downflow at low voidages that bubbles favored the pipe center, which would account for the high values of $C_o$ found at voidages less than 10%. At higher voidages a more even distribution would lead to lower values of the profile constant.

The mean values of $C_o$ in upflow found here were a little lower than those given in the literature, but it should be noted that no previous study of bubbly flow in a pipe of such large diameter has been reported. It may be supposed that as pipe diameter increases, bubble behavior may become less symmetrical and less predictable. This trend is certainly born out by the complex behavior of rising bubbles in large diameter bubble columns (Lockett and Kirkpatrick, 1975).

## NOTATION

| | |
|---|---|
| $C_o$ | = Zuber and Findlay profile constant, 0 |
| $C_1,C_2$ | = constants for profile interaction in Eq. 4, 0 |
| $D$ | = tube diameter, m |
| $g$ | = acceleration due to gravity, m/s$^2$ |
| $n$ | = exponent in Eq. 3, 0 |
| $U$ | = velocity, m/s |
| $U_{gl}$ | = local slip velocity between bubble and liquid, m/s |
| $U_{gm}$ | = drift velocity, m/s |
| $U_b$ | = rise velocity of bubble in infinite continuum, m/s |
| $W$ | = flux or superficial velocity, m/s |
| $\epsilon$ | = gas voidage, 0 |
| $\rho$ | = liquid density, kg/m$^3$ |
| $\sigma$ | = surface tension, N/m |

### Subscripts

| | |
|---|---|
| av. | = averaged over pipe cross section |
| $g$ | = refers to gas phase |
| $l$ | = refers to liquid phase |
| $t$ | = total flow property |

## LITERATURE CITED


Ardron, K. H., and P. C. Hall, "Prediction of Void Fraction in Low Velocity Steam-Water Flow," *ASME J Heat Transfer*, **102**, 3 (1980).

Bhaga, D., and M. E. Weber, "Holdup in Vertical Two and Three Phase Flow," *Can. J. Chem. Eng.*, **50**, 323 (1972).

Brown, F. C., A. Gomezplata and J. D. Price, "A Model to Predict Void Fraction in Two Phase Flow," *Chem. Eng. Sci.*, **24**, 1483 (1969).

Brodkey, R. S., *The Phenomena of Fluid Motions*, Addison Wesley, Reading, MA (1967).

Clark, N. N., and R. L. C. Flemmer, "Narrow Bore Deep Shaft Reactors," *CHEMSA*, **9**, 62 (1983).

———, "On Vertical Downward Two Phase Flow," *Chem. Eng. Sci.*, **39**, 170 (1984a).

———, "A technique for Synchronising Valves and Determining Bubble Rise Velocities in Two Phase Flow," Paper WA-84-FE-11, ASME Winter Meeting, New Orleans (1984b).

Collins, O. C., and M. D. Elder, "Experience in Operating the Deep Shaft Activated Sludge Process," *Water Pollution Control*, **79**, 272 (1980).

Cox, G. C., et al., "Use of the Deep Shaft Process in Uprating and Extending Existing Sewage Treatment Works," *Water Pollution Control*, **79**, 70 (1980).

Galaup, J.-P., and D.-M. Delhaye, "Utilization de Sondes Optiques Miniatures en Ecoulement Diphasique Gaz-Liquide," *La Houille Blanche*, **1-1976**, 17 (1976).

Govier, G. W., and K. Aziz, *The Flow of Complex Mixtures in Pipes*, Van Nostrand Reinhold, New York (1972).

Harmathy, T. Z., "Velocity of Large Drops and Bubbles in Media of Restricted or Infinite Extent," *AIChE J.*, **6**, 281 (1960).

Herringe, R. A., and M. R. Davis, "Structural Development of Gas-Liquid Flows," *J. Fluid. Mech.*, **73**, 97 (1976).

Ishii, M., and M. A. Grolmes, "Constitutive Equation for One Dimensional Drift Velocity of Dispersed Two Phase Flow," in *Two Phase Transport and Reactor Safety*, Veziroglu and Kakac, eds., Hemisphere, Washington, D.C. (1978).

Kubota, H., Y. Hosono, and K. Fujie, "Characteristic Evaluations of the ICI Air Lift Type Deep Shaft Aerator," *J. Chem. Eng. Japan*, **11**, 319 (1978).

Lockett, M. J., and R. D. Kirkpatrick, "Ideal Bubbly Flow and Actual Flow in Bubble Columns," *Trans. Inst. Chem. Engrs.*, **53**, 267 (1975).

Lorenzi, A., and G. Sotgia, "Comparative Investigation of Some Characteristic Quantities of Two Phase Cocurrent Upward and Downward Flow," in *Two Phase Transport and Reactor Safety*, Veziroglu and Kakac, eds., Hemisphere, Washington, D.C. (1978).

Martin, C. S., "Vertically Downward Two Phase Slug Flow," *ASME J. Fluids Eng.*, **98**, 715 (1976).

Nassos, G. P., and S. G. Bankoff, "Slip Velocity Ratios in an Air-Water System under Steady State and Transient Conditions," *Chem. Eng. Sci.*, **22**, 661 (1967).

Oshinowo, T., and M. E. Charles, "Vertical Two Phase Flow," *Can. J. Chem. Eng.*, **52**, 25 and 438 (1974).

Petrick, M., and A. A. Kurdika, "On the Relationship Between the Phase Distribution and Relative Velocities in Two Phase Flow," *AIChE Proc. 3rd Intern. Heat Transfer Conf.*, **4**, 184 (1966).

Serizawa, A., I. Katoka, and I. Michiyoshi, "Turbulence Structure of Air-Water Bubble Flow," *Intern. J. Multiphase Flow*, **2**, 235 (1975).

Wallis, G. B., *One Dimensional Two Phase Flow*, McGraw-Hill, New York (1969).

Zuber, N., and J. A. Findlay, General Electric Report GEAP-4592 (1964).

———, "Average Volumetric Concentration in Two Phase Flow Systems," *ASME J. Heat Transfer*, **87**, 453 (1965).

Zuber, N., F. W. Staub, G. Bijwaard, and P. G. Kroeger, General Electric Report GEAP-5417 (1967).


*Manuscript received Nov. 17, 1983; revision received Dec. 7, 1983, and accepted Dec. 7.*